Introduction to Econometrics
Chapter 2

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2 The simple regression model: estimation and properties

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2.1 Some definitions in the simple regression model

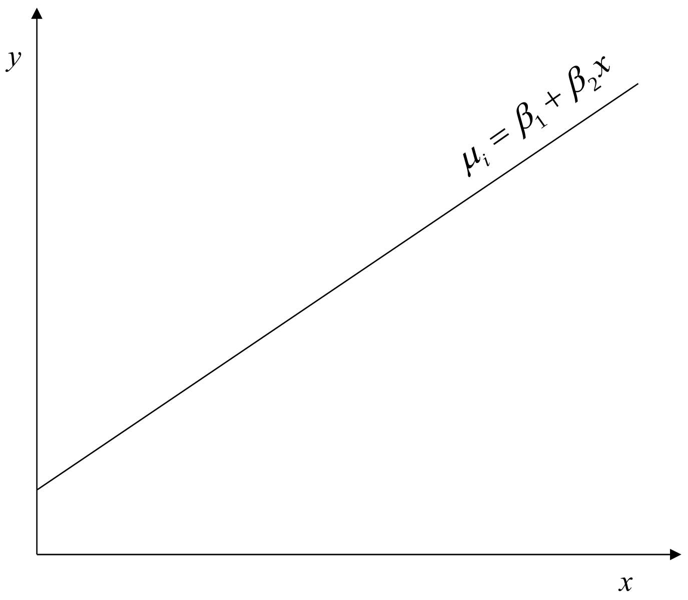


FIGURE 2.1. The population regression function. (PRF)

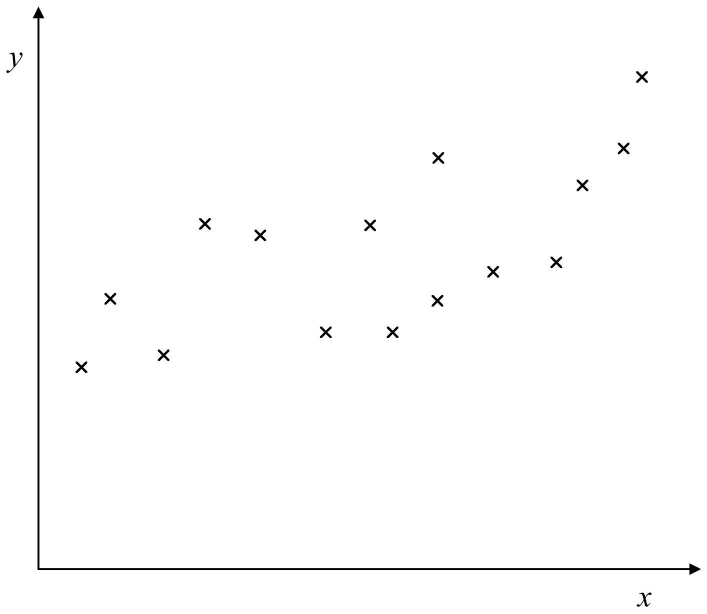


FIGURE 2.2. The scatter diagram..

2.1 Some definitions in the simple regression model

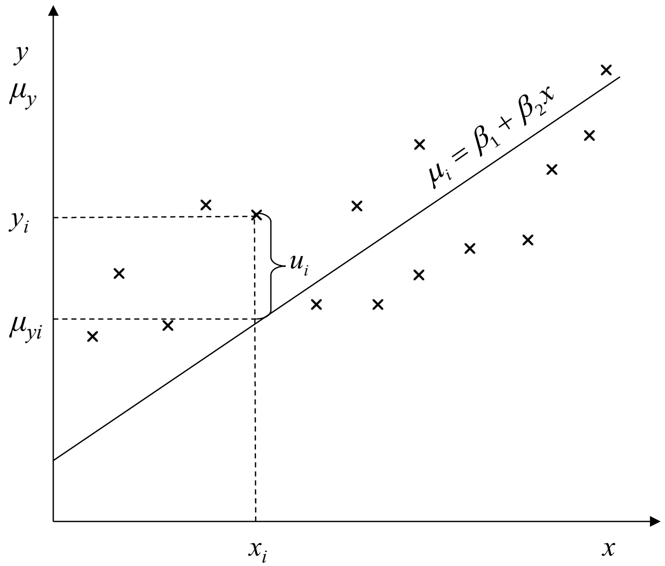


FIGURE 2.3. The population regression function and the scatter diagram.

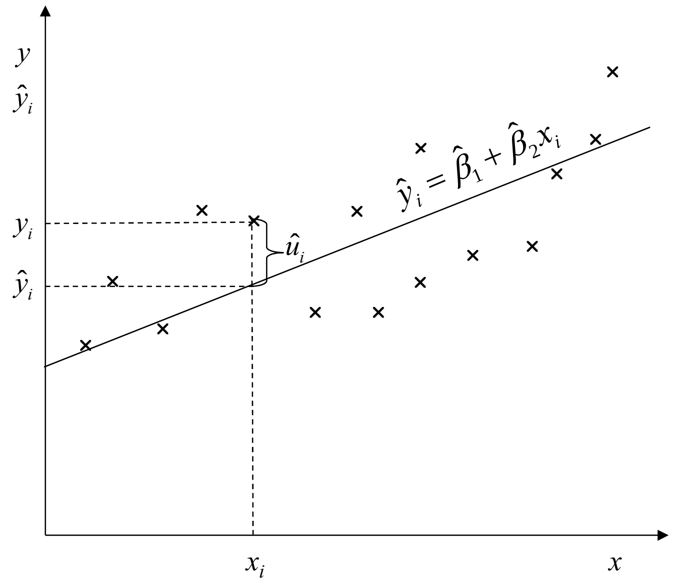


FIGURE 2.4. The sample regression function and the scatter diagram.

2.2 Obtaining the Ordinary Least Squares Estimates

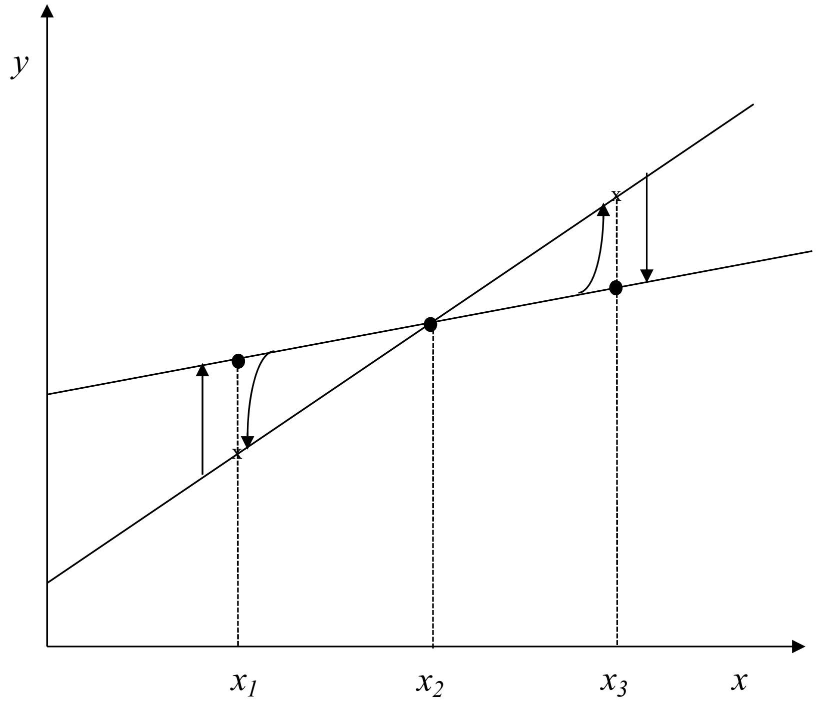


FIGURE 2.5. The problems of criterion 1.

2.2 Obtaining the Ordinary Least Squares Estimates

EXAMPLE 2.1 Estimation of the consumption function

$$cons = \beta_1 + \beta_2 inc + u_i$$

TABLE 2.1. Data and calculations to estimate the consumption function.

Observ.	$cons_i$	inc_i	$cons_i \times inc_i$	inc_i^2	$cons_i - \overline{cons}$	$inc_i - \overline{inc}$	$(cons_i - \overline{cons}) \times (inc_i - \overline{inc})$	$(inc_i - \overline{inc})^2$
1	5	6	30	36	-4	-5	20	25
2	7	9	63	81	-2	-2	4	4
3	8	10	80	100	-1	-1	1	1
4	10	12	120	144	1	1	1	1
5	11	13	143	169	2	2	4	4
6	13	16	208	256	4	5	20	25
Sums	**54**	**66**	**644**	**786**	**0**	**0**	**50**	**60**

$$\overline{cons} = \frac{54}{6} = 9 \qquad \overline{inc} = \frac{66}{6} = 11 \qquad \text{(2-17)}: \hat{\beta}_2 = \frac{644 - 9 \times 66}{786 - 11 \times 66} = 0.8\widehat{3}$$

$$\text{(2-18)}: \hat{\beta}_2 = \frac{50}{60} = 0.8\widehat{3} \qquad \hat{\beta}_1 = 9 - 0.8\widehat{3} \times 11 = -0.1\widehat{6}$$

EXAMPLE 2.2 Fulfilling algebraic implications and calculating R^2 in the consumption function

$$TSS = 42 \qquad ESS = 41.67 \qquad RSS = 42 - 41.67 = 0.33 \qquad R^2 = \frac{41.67}{42} = 0.992$$

or, alternatively,

$$R^2 = 1 - \frac{0.33}{42} = 0.992$$

TABLE 2.2. Data and calculations to estimate the consumption function.

Observ.	$\widehat{cons}_i$	$\hat{u}_i$	$\hat{u}_i \times inc_i$	$\widehat{cons}_i \times \hat{u}_i$	$cons_i^2$	$(cons_i - \overline{cons})^2$	$\widehat{cons}_i^2$	$(\widehat{cons}_i - \overline{\widehat{cons}})^2$
1	4.83	0.17	1	0.81	25	16	23.36	17.36
2	7.33	-0.33	-3	-2.44	49	4	53.78	2.78
3	8.17	-0.17	-1.67	-1.36	64	1	66.69	0.69
4	9.83	0.17	2	1.64	100	1	96.69	0.69
5	10.67	0.33	4.33	3.56	121	4	113.78	2.78
6	13.17	-0.17	-2.67	-2.19	169	16	173.36	17.36
	54	**0**	**0**	**0**	**528**	**42**	**527.67**	**41.67**

2.3 Some characteristics of *OLS* estimators

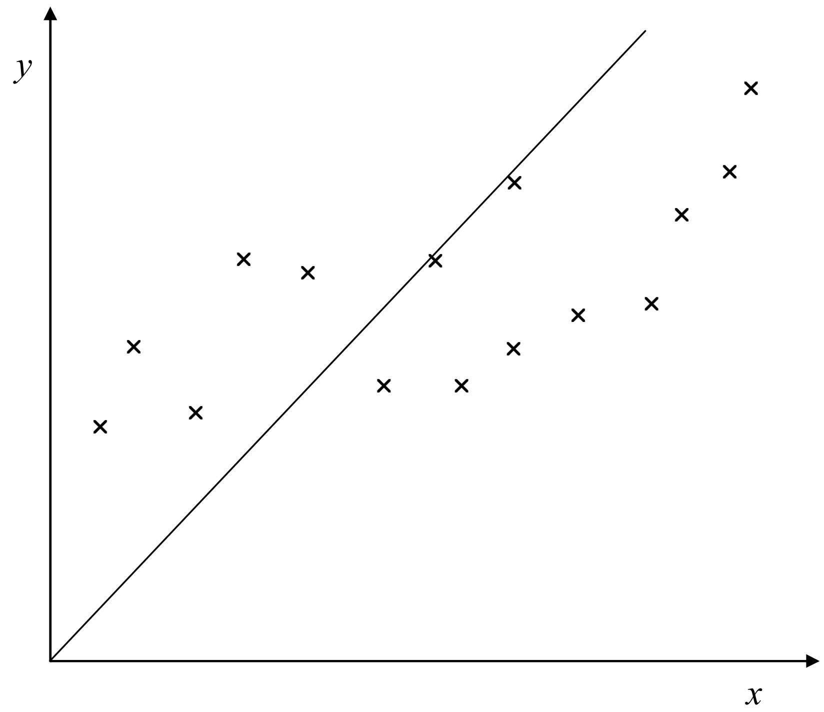


FIGURE 2.6. A regression through the origin.

2.4 Units of measurement and functional form

EXAMPLE 2.3

$$\text{(2-39)}: \widehat{cons}_i = 0.2 + 0.85 \times inc_i$$

$$ince = inc \times 1000$$

$$\widehat{cons}_i = 0.2 + 0.00085 \times ince_i$$

EXAMPLE 2.4

$$conse = cons \times 1000$$

$$\widehat{conse}_i = 200 + 850 \times inc_i$$

2.4 Units of measurement and functional form

EXAMPLE 2.5

$$\overline{inc} = 20 \qquad incd_i = inc_i - \overline{inc}$$

$$\widehat{cons}_i = (0.2 + 0.85 \times 20) + 0.85 \times (inc_i - 20) = 17.2 + 0.85 \times incd_i$$

EXAMPLE 2.6

$$\overline{cons} = 15 \qquad consd_i = cons_i - \overline{cons}$$

$$\widehat{cons}_i - 15 = 0.2 - 15 + 0.85 \times inc_i$$

$$\widehat{consd}_i = -14.8 + 0.85 \times inc_i$$

2.4 Units of measurement and functional form

TABLE 2.3. Examples of proportional change and change in logarithms.

x_1	202	210	220	240	300
x_0	200	200	200	200	200
Proportional change in %	1%	5,0%	10,0%	20,0%	50,0%
Change in logarithms in %	1%	4,9%	9,5%	18,2%	40,5%

2.4 Units of measurement and functional form

EXAMPLE 2.7 Quantity sold of coffee as a function of its price. Linear model (file coffee1)

$$coffqty = \beta_1 + \beta_2 coffpric + u$$

$$\widehat{coffqty} = 774.9 - 693.33 coffpric \quad R^2 = 0.95 \quad n = 12$$

TABLE 2.4. Data on quantities and prices of coffee.

week	*coffpric*	*coffqty*
1	1.00	89
2	1.00	86
3	1.00	74
4	1.00	79
5	1.00	68
6	1.00	84
7	0.95	139
8	0.95	122
9	0.95	102
10	0.85	186
11	0.85	179
12	0.85	187

2.4 Units of measurement and functional form

EXAMPLE 2.8 Explaining market capitalization of Spanish banks. Linear model (file bolmad95)

$$\widehat{marktval} = 29.42 + 1.219 bookval$$
$$R^2 = 0.836 \quad n = 20$$

EXAMPLE 2.9 Quantity sold of coffee as a function of its price. Log- log model (Continuation example 2.7) (file coffee1)

$$\ln(\widehat{coffqty}) = 4.415 - 5.132 \ln(coffpric)$$
$$R^2 = 0.90 \quad n = 12$$

EXAMPLE 2.10 Explaining market capitalization of Spanish banks. Log-log model (Continuation example 2.8) (file bolmad95)

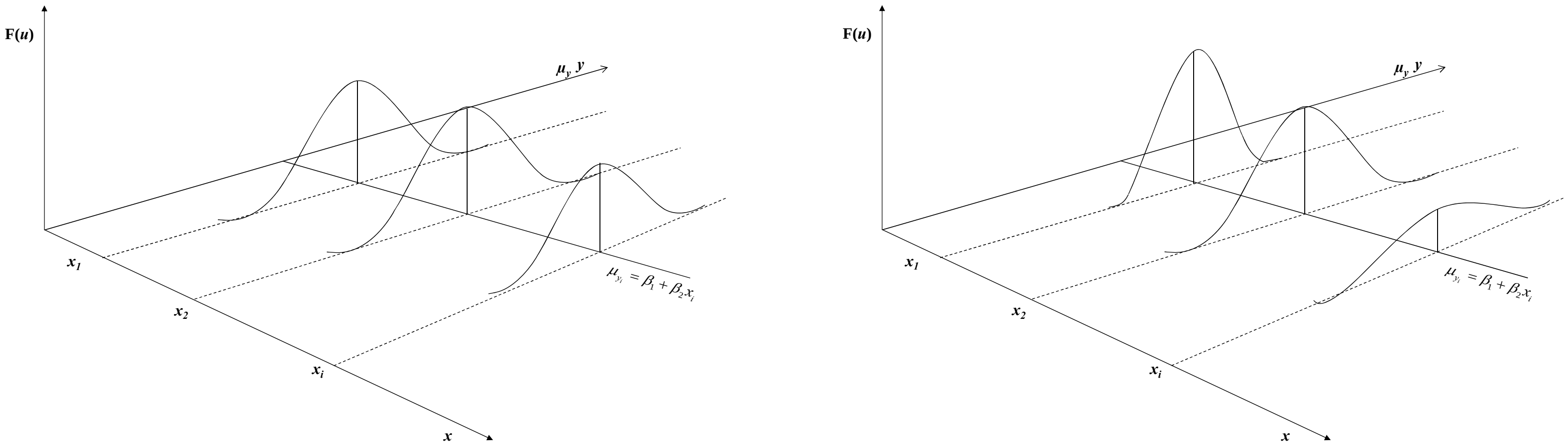
$$\widehat{\ln(marktval)} = 0.6756 + 0.938\ln(bookval)$$

$$R^2 = 0.928 \quad n = 20$$

TABLE 2.5. Interpretation of in different models..

Model	If x increases by	then y will increase by
linear	1 unit	$\hat{\beta}_2$ units
linear-log	1%	$(\hat{\beta}_2/100)$ units
log-linear	1 unit	$(100\hat{\beta}_2)\%$
log-log	1%	$\hat{\beta}_2\%$

2.5 Assumptions and statistical properties of *OLS*



a) **b)**

FIGURE 2. 7. Random disturbances:
a) homoscedastic; b) heteroskedastic.

2.5 Assumptions and statistical properties of *OLS*

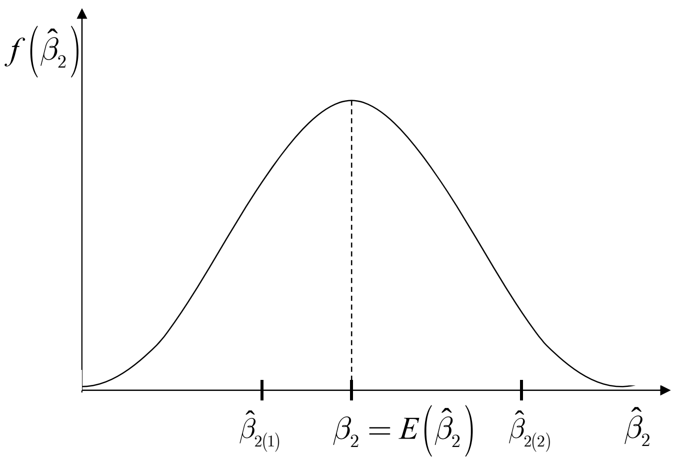


FIGURE 2.8. Unbiased estimator.

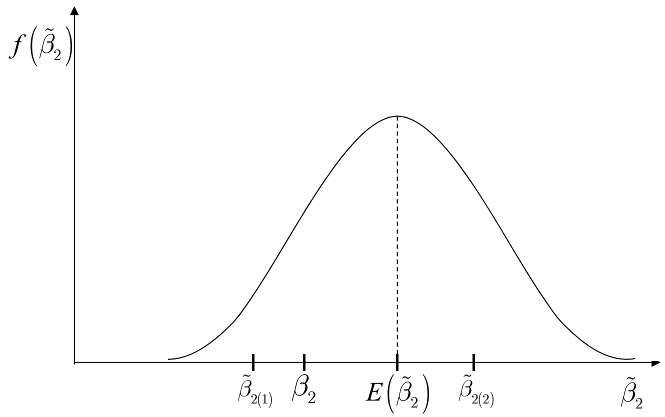


FIGURE 2.9. Biased estimator.

2.5 Assumptions and statistical properties of *OLS*

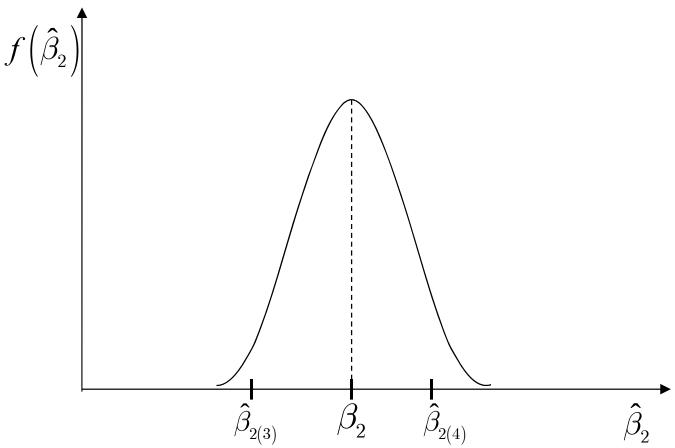


FIGURE 2.10. Estimator with small variance.

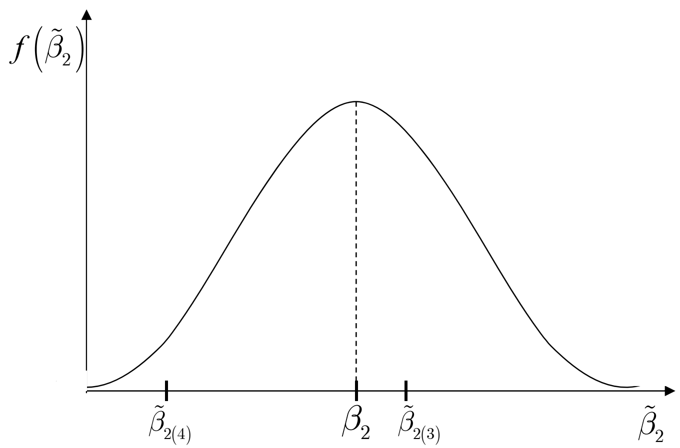


FIGURE 2.11. Estimator with big variance.

2.5 Assumptions and statistical properties of *OLS*

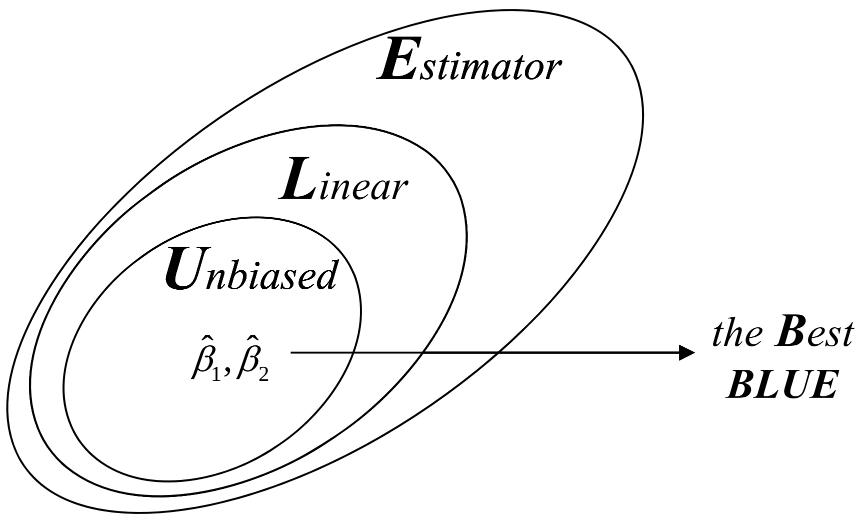


FIGURE 2.12. The *OLS* estimator is the BLUE.

2.5 Assumptions and statistical properties of *OLS*

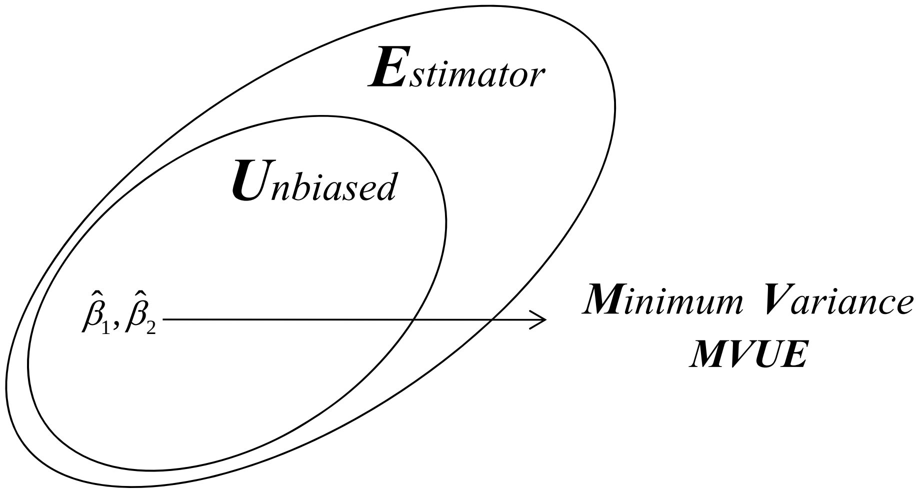


FIGURE 2.13. The *OLS* estimator is the MVUE.

Annex 2.1 Case study: Engel curve for demand of dairy products (file demand)

TABLE 2.6 Expenditure in dairy products (dairy), disposable income (inc) in terms per capita. Unit: euros per month. n=40

household	*dairy*	*inc*	*household*	*dairy*	*inc*
1	8.87	1.25	21	16.2	2.1
2	6.59	985	22	10.39	1.47
3	11.46	2.175	23	13.5	1.225
4	15.07	1.025	24	8.5	1.38
5	15.6	1.69	25	19.77	2.45
6	6.71	670	26	9.69	910
7	10.02	1.6	27	7.9	690
8	7.41	940	28	10.15	1.45
9	11.52	1.73	29	13.82	2.275
10	7.47	640	30	13.74	1.62
11	6.73	860	31	4.91	740
12	8.05	960	32	20.99	1.125
13	11.03	1.575	33	20.06	1.335
14	10.11	1.23	34	18.93	2.875
15	18.65	2.19	35	13.19	1.68
16	10.3	1.58	36	5.86	870
17	15.3	2.3	37	7.43	1.62
18	13.75	1.72	38	7.15	960
19	11.49	850	39	9.1	1.125
20	6.69	780	40	15.31	1.875

Annex 2.1 Case study: Engel curve for demand of dairy products

Linear model

$$dairy = \beta_1 + \beta_2 inc + u$$

$$\frac{d\ dairy}{d\ inc} = \beta_2$$

$$\varepsilon_{dairy/inc}^{linear} = \frac{d\ dairy}{d\ inc}\frac{inc}{dairy} = \beta_2 \frac{inc}{dairy}$$

$$\widehat{dairy} = 4.012 + 0.005288 \times inc \qquad R^2 = 0.4584$$

Annex 2.1 Case study: Engel curve for demand of dairy products

Inverse model

$$dairy = \beta_1 + \beta_2 \frac{1}{inc} + u$$

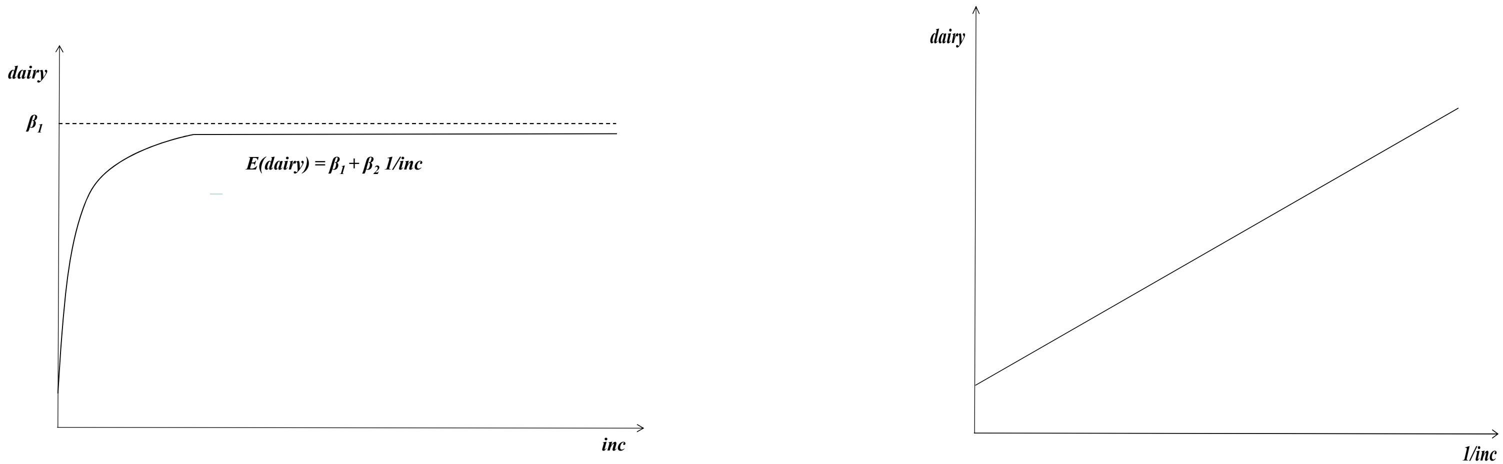


FIGURE 2.14. The inverse model.

$$\frac{d\ dairy}{d\ inc} = -\beta_2 \frac{1}{(inc)^2}$$

$$\varepsilon_{dairy/inc}^{inv} = \frac{d\ dairy}{d\ inc}\frac{inc}{dairy} = -\beta_2 \frac{1}{inc \times dairy}$$

$$\widehat{dairy} = 18.652 - 8702\frac{1}{inc} \qquad R^2 = 0.4281$$

Annex 2.1 Case study: Engel curve for demand of dairy products

Linear-log model

$$dairy = \beta_1 + \beta_2 \ln(inc) + u$$

$$\frac{d\ dairy}{d\ inc} = \frac{d\ dairy}{d\ inc}\frac{inc}{inc} = \frac{d\ dairy}{d\ln(inc)}\frac{1}{inc} = \beta_2\frac{1}{inc}$$

$$\varepsilon_{dairy/inc}^{lin\text{-}\log} = \frac{d\ dairy}{d\ inc}\frac{inc}{dairy} = \frac{d\ dairy}{d\ \ln(inc)}\frac{1}{dairy} = \beta_2\frac{1}{dairy}$$

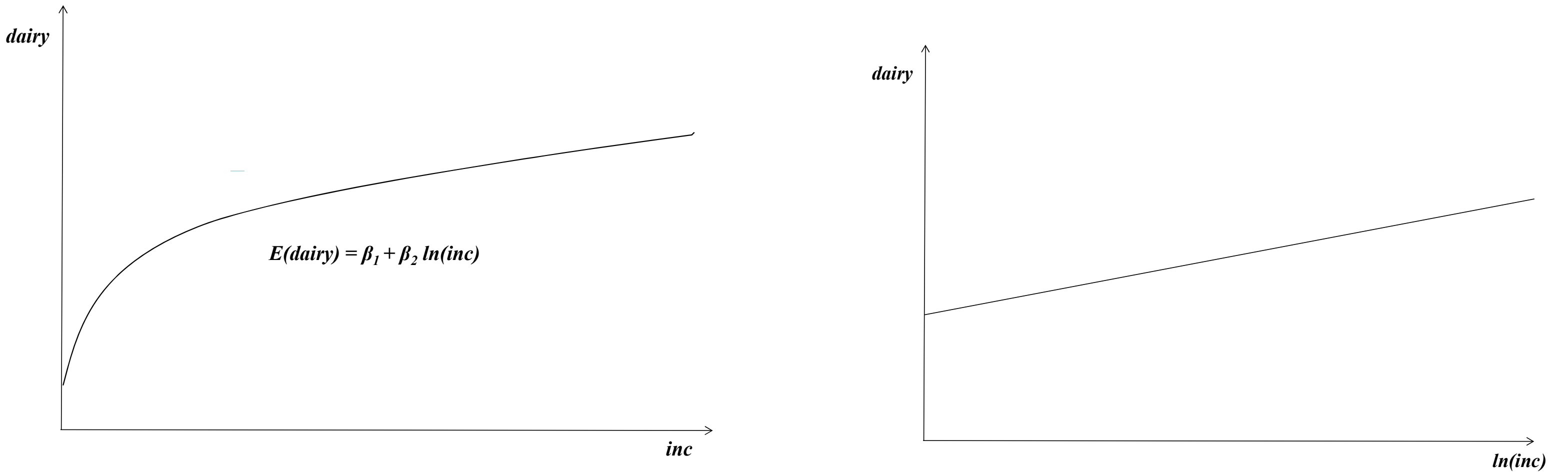


FIGURE 2.15. The linear log model.

$$\widehat{dairy} = -41.623 + 7.399 \times \ln(inc) \quad R^2 = 0.4567$$

Annex 2.1 Case study: Engel curve for demand of dairy products

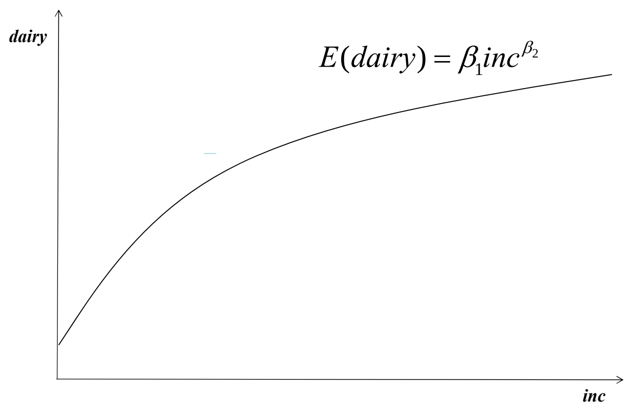
Log-log model or potential model

$$dairy = e^{\beta_1} inc^{\beta_2} e^{u}$$

$$\ln(dairy) = \beta_1 + \beta_2 \ln(inc) + u$$

$$\frac{d\ dairy}{d\ inc} = \beta_2 \frac{dairy}{inc}$$

$$\varepsilon_{dairy/inc}^{log\text{-}log} = \frac{d\ dairy}{d\ inc}\frac{inc}{dairy} = \frac{d\ \ln(dairy)}{d\ \ln(inc)} = \beta_2$$



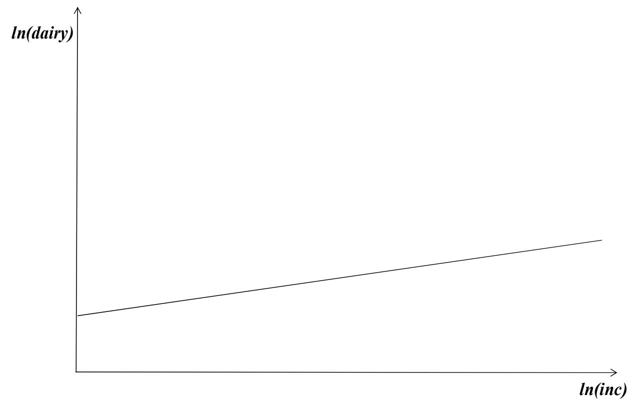


FIGURE 2.16. The log log model.

$$\widehat{\ln(dairy)} = -2.556 + 0.6866 \times \ln(inc) \quad R^2 = 0.5190$$

Annex 2.1 Case study: Engel curve for demand of dairy products

Log-linear or exponential model

$$dairy = \exp(\beta_1 + \beta_2 inc + u)$$

$$\ln(dairy) = \beta_1 + \beta_2 inc + u$$

$$\frac{d\ dairy}{d\ inc} = \beta_2 dairy$$

$$\varepsilon^{exp}_{dairy/inc} = \frac{d\ dairy}{d\ inc}\frac{inc}{dairy} = \frac{d\ \ln(dairy)}{d\ inc} inc = \beta_2 inc$$

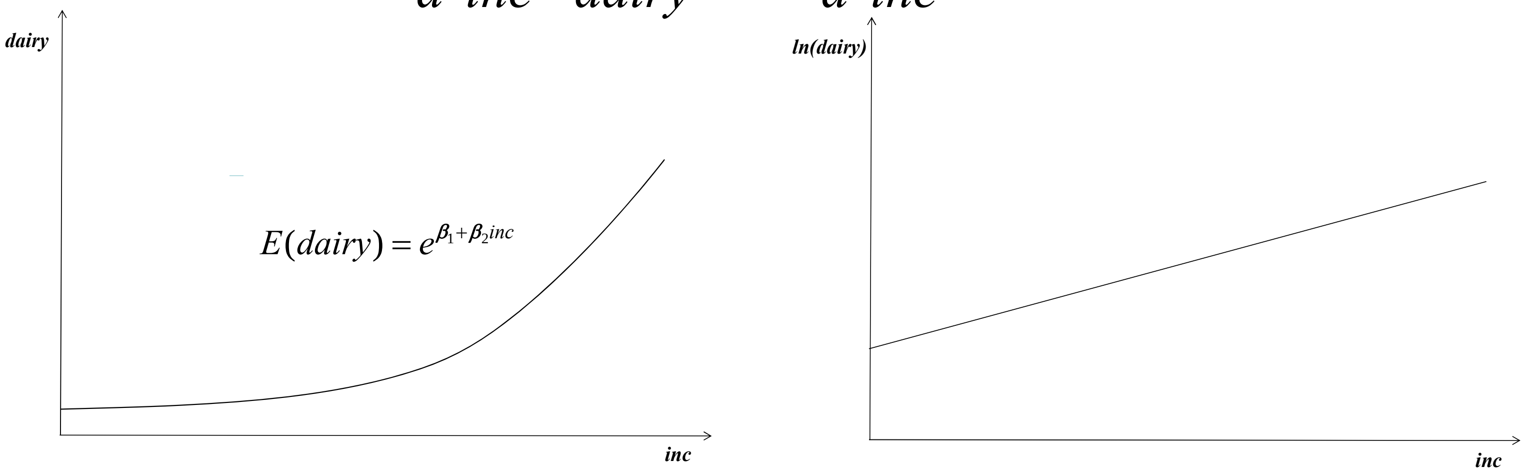


FIGURE 2.17. The log linear model.

$$\widehat{\ln(dairy)} = 1.694 + 0.00048 \times inc \qquad R^2 = 0.4978$$

Annex 2.1 Case study: Engel curve for demand of dairy products

Inverse exponential model

$$dairy = \exp(\beta_1 + \beta_2 \frac{1}{inc} + u)$$

$$\ln(dairy) = \beta_1 + \beta_2 \frac{1}{inc} + u$$

$$\frac{d\ dairy}{d\ inc} = -\beta_2 \frac{dairy}{(inc)^2}$$

$$\varepsilon_{dairy/inc}^{invexp} = \frac{d\ dairy}{d\ inc}\frac{inc}{dairy} = \frac{d\ \ln(dairy)}{d\ inc} inc = -\beta_2 \frac{1}{inc}$$

$$\widehat{\ln(dairy)} = 3.049 - 822.02 \frac{1}{inc} \qquad R^2 = 0.5040$$

Annex 2.1 Case study: Engel curve for demand of dairy products

TABLE 2.7. Marginal propensity, expenditure/income elasticity and R^2 in the fitted models.

Model	*Marginal propensity*	*Elasticity*	R^2
Linear	$\hat{\beta}_2 = 0.0053$	$\hat{\beta}_2 \dfrac{\overline{inc}}{\overline{dairy}} = 0.6505$	0.4440
Inverse	$-\hat{\beta}_2 \dfrac{1}{\left[\overline{inc}\right]^2} = 0.0044$	$-\hat{\beta}_2 \dfrac{1}{\overline{dairy} \times \overline{inc}} = 0.5361$	0.4279
Linear-log	$\hat{\beta}_2 \dfrac{1}{\overline{inc}} = 0.0052$	$\hat{\beta}_2 \dfrac{1}{\overline{dairy}} = 0.6441$	0.4566
Log-log	$\hat{\beta}_2 \dfrac{\overline{dairy}}{\overline{inc}} = 0.0056$	$\hat{\beta}_2 = 0.6864$	0.5188
Log-linear	$\hat{\beta}_2 \times \overline{dairy} = 0.0055$	$\hat{\beta}_2 \times \overline{inc} = 0.6783$	0.4976
Inverse-log	$-\hat{\beta}_2 \dfrac{\overline{dairy}}{\left[\overline{inc}\right]^2} = 0.0047$	$-\hat{\beta}_2 \dfrac{1}{\overline{inc}} = 0.5815$	0.5038